Modular Theory for Operator Algebra in Bounded Region of Space-Time and Quantum Entanglement

Daisuke Ida, Takahiro Okamoto, and Miyuki Saito

Department of Physics, Gakushuin University, Tokyo 171-8588, Japan.

(November 16, 2012)

Abstract. We consider the quantum state seen by an observer in the diamond-shaped region, which is a globally hyperbolic open submanifold of the Minkowski space-time. It is known from the operator-algebraic argument that the vacuum state of the quantum field transforming covariantly under the conformal group looks like a thermal state on the von Neumann algebra generated by the field operators on the diamond-shaped region of the Minkowski space-time. Here, we find that such a state can in fact be identified with a certain entangled quantum state. By doing this, we obtain the thermodynamic quantities such as the Casimir energy and the von Neumann entropy of the thermal state in the diamond-shaped region. We further speculate on a possible information-theoretic interpretation of the entropy in terms of the probability density functions naturally determined from the Tomita-Takesaki modular flow in the diamond-shaped region.

We often regard the quantum state of a field on the space-time as being a pure state that has the zero von Neumann entropy. Of course, this does not imply that an observer always has a perfect knowledge of the quantum field. Rather, each observer would not be able to distinguish it from a certain mixed state, and the identified mixed state would in general depend on the observer's trajectory and the measuring means available. Thus, each observer has his own nonzero von Neumann entropy for the quantum states of the field.

For example, let us consider an observer with a finite lifetime whose world-line is a timelike segment in the space-time bounded by future and past end points, and the measurements of the quantum field by him in terms of an apparatus located at each space-time point. When this observer sends a command to a remote measuring apparatus, the apparatus immediately performs a measurement of the quantum field on the corresponding space-time point and the result is returned to the observer. If this is the only way for the observer to measure the quantum state of the field, the set of points from which the observer can get the information is the intersection of the chronological future and the chronological past of the observer's world-line, which we call, for obvious reasons, the "diamond region" associated with the observer. The limitation of the observed region would cause the loss of the information on the quantum state of the field. This can be heuristically understood from general considerations as follows.

In general, a quantum measurement can be reduced to the evaluation of the expectation value of a non-negative self-adjoint operator belonging to a C^*-algebra $\mathscr{A}$. In the quantum field theory, the corresponding C^*-algebra $\mathscr{A}$ might be regarded as the von Neumann algebra $\mathscr{A}(M)$ constructed from the field operators on the space-time M. (Though the polynomial $*$-algebra generated by field operators is not a von Neumann algebra, for the field operators are unbounded, one can define a von Neumann algebra $\mathscr{A}(O)$ constructed from field operators on the open

submanifold O of M, if O is M itself, a diamond region, a so-called Rindler wedge, or their image under a Poincaré transformation [1, 2]. More precisely, the von Neumann algebra $\mathscr{A}(O)$ is the double commutant of the C^*-algebra generated by the projection operators composing the field operators smeared by test functions with support in O.)

However, not all the projection operators in $\mathscr{A}(M)$ are available for every observer. Rather, the available projection operators, or more generally non-negative self-adjoint operators, generate a proper von Neumann subalgebra of $\mathscr{A}(M)$, which would be regarded as the algebra of physical quantities for the observer. For an observer with a finite lifetime, the corresponding von Neumann subalgebra of physical quantities would be $\mathscr{A}(O)$, where O is the diamond region associated with the observer.

On the other hand, a quantum state $\omega : \mathscr{A} \to \boldsymbol{C}$ on a C^*-algebra $\mathscr{A}$ is a pure state if and only if the GNS representation of $\mathscr{A}$ associated with the quantum state ω is irreducible. However, the GNS representation of its C^*-subalgebra $\mathscr{A}'$ associated with the restriction of ω to $\mathscr{A}'$ is not always irreducible. If it is reducible, the quantum state ω is indistinguishable from a certain mixed state in terms of any quantum measurements solely of the operators in $\mathscr{A}'$.

Hence, an observer with a finite lifetime would perceive a certain mixed state. Then, how does the vacuum state in the Minkowski space-time look like for the observer with the finite lifetime?

In the case of the conformally invariant Hermitian scalar field, Martinetti and Rovelli [3] conclude that such an observer will see a certain thermal state. Their reasoning is based on the conformal invariance of the vacuum state and the conformal equivalence between the diamond region and the Rindler wedge. The outline of their argument is as follows.

Let W be the Rindler wedge, which is the open submanifold of the n-dimensional Minkowski space-time ($n \geq 2$) specified by $x^1 > |x^0|$ in terms of the standard time coordinate x^0 and one of the standard spatial coordinates x^1 in the Minkowski space-time. The Rindler wedge W is globally static in the sense that the Lorentz boost generated by the Killing vector field $x^1\partial_0 + x^0\partial_1$ acts isometrically on W. A uniformly accelerated observer following an orbit of the Lorentz boost in the Poincaré invariant vacuum state would find himself apparently in a thermal bath with the temperature proportional to the proper acceleration. This is well known as the Unruh effect [4].

One of rigorous explanations of the Unruh effect is given by the Bisognano-Wichmann theorem [5]. This theorem shows that the von Neumann algebra $\mathscr{A}(W)$ gives in an essential way an example of the application of the Tomita-Takesaki modular theory [6] of operator algebras. According to the Tomita's fundamental theorem in the modular theory, given a von Neumann algebra $\mathscr{A}$ acting on a Hilbert space H, and a cyclic and separating vector $|\Omega\rangle \in H$, there uniquely exists the one-parameter group of automorphism $\{\sigma_s\}$ acting on $\mathscr{A}$, which is called the modular flow. Furthermore, the modular flow is subject to the Kubo-Martin-Schwinger (KMS) condition with respect to the vector state corresponding to $|\Omega\rangle$, which means that $|\Omega\rangle$ is identified with a thermal state. The Bisognano-Wichmann theorem states that in the case of $\mathscr{A} = \mathscr{A}(W)$, $|\Omega\rangle$ corresponds to the Poincaré invariant vacuum, and hence the vacuum is subject to the KMS condition, where the

generator of the Lorentz boost plays a role of the Hamiltonian. Thus, the modular flow here can be seen as the geometric flow generating the time translation in W.

A relativistic quantum field in the Minkowski space-time is often assumed to transform covariantly under the Poincaré group [7]. If we further require the covariance under the conformal group, and the conformal invariance of the vacuum state, we can, in a sense, map the geometric modular flow in the Rindler wedge W to that in the conformal image of W. (Though in the case of $n = 2$, there is no vacuum state invariant under the conformal group, it is sufficient to consider a state invariant under the projective conformal group, which is the subgroup generated by the dilatations, the special conformal transformations and the Poincaré transformations.)

In fact, Hislop and Longo show that for the quantum field in the diamond region O, the conformally invariant vacuum is subject to the KMS condition [8], which relies on the conformal equivalence between the Rindler wedge and the diamond region.

Martinetti and Rovelli interpret the modular flow as determining the "thermal time" in O, and this leads to the notion of the "diamond temperature" which is the proper temperature for the observer following the modular flow [3]. The relevant observer in O is the inertial observer or the uniformly accelerated observer with the finite lifetime. A remarkable point here is that even an inertial observer may perceive a nonzero temperature. Another feature of the diamond temperature is that it in general diverges around the future and past end points of the observer's world-line.

It is not clear whether the behavior of the diamond temperature as above is universal one or whether it is peculiar to the operator-algebraic method. Hence, we would like to verify the diamond temperature in terms of the standard method [9] via the determination of the Bogoliubov transformation between different Fock representations. We will see that it gives the same temperature as that derived by Martinetti and Rovelli. Then, we discuss the thermodynamic quantities such as the Casimir energy and the quantum entanglement entropy for the observer with a finite lifetime based on the standard quantum field theory. We further introduce the probability density function naturally determined by the modular flow in the diamond region, and attempt to give the information-theoretic interpretation of the entropy of the diamond region.

In this letter, we consider the free massless Hermitian scalar field in the 2-dimensional Minkowski space-time M, which transforms covariantly under the projective conformal group. We use the natural unit system in which $c = \hbar = 1$. The diamond region O is specified by $|t| + |x| < L$ with a length parameter L, when the Lorentzian metric is written as $ds^2 = -dt^2 + dx^2$ (Fig. 1).

The modular flow in O coincides with the geometric flow generated by the conformal Killing vector field, which is timelike in O. This conformal Killing vector field naturally defines the positive frequency modes of the Hermitian scalar field for observers following the modular flow. In fact, we define the positive frequency modes as the conformal image of the positive frequency solutions defined on the Rindler wedge W, under the conformal diffeomorphism: $W \to O$, which pushes forward the timelike Killing vector field in W to the conformal Killing vector field in O. More precisely, if the Lorentzian metric $g^O_{\mu\nu}$ in O is conformally equivalent

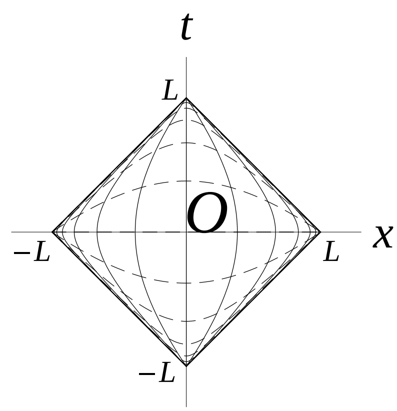

FIGURE 1. The diamond region O of the Minkowski space-time is depicted. The solid curve denotes constant X and the dashed curve represents constant T [see Eq. (3)].

to the Lorentzian metric $g^W_{\mu\nu}$ in W as $g^O_{\mu\nu} = C^2 g^W_{\mu\nu}$, and ξ^μ is the timelike Killing vector field with respect to $g^W_{\mu\nu}$, then ξ^μ is the conformal Killing vector field with respect to $g^O_{\mu\nu}$. The positive frequency mode χ^O_ω in O is required to satisfy the eigenvalue equation on O

$$\xi^\mu \partial_\mu \chi^O_\omega = -i\omega \chi^O_\omega$$

for $\omega > 0$.

The null coordinates $U^\pm \in \boldsymbol{R}^1$ covering O are defined by

$$u^\pm = L \tanh(U^\pm / L),$$

where $u^\pm = t \pm x$ are the Minkowski null coordinates. Then, the positive frequency modes in O are subject to

$$\left(\frac{\partial}{\partial U^+} + \frac{\partial}{\partial U^-} \right) \chi^O_\omega = -i\omega \chi^O_\omega, \;\; \frac{\partial^2}{\partial U^+ \partial U^-} \chi^O_\omega = 0.$$

Therefore, the positive frequency mode in O consists of

$$\chi^{O\pm}_\omega = \frac{1}{\sqrt{4\pi\omega}} \exp(-i\omega U^\pm).$$

For the later convenience, we introduce the mode functions

$$\chi^\pm_\omega = \frac{1}{\sqrt{4\pi\omega}} \exp(-i\omega U^\pm(u^\pm)) \theta(L - |u^\pm|)$$

as extension of $\chi^{O\pm}_\omega$ to M, where we call χ^+_ω the ingoing mode, and χ^-_ω the outgoing mode, and these mode functions are normalized with respect to the Klein-Gordon inner product.

On the other hand, by continuing analytically the positive frequency modes $\chi_\omega^{O\pm}$ to M, we obtain

$$
\begin{aligned}
\widetilde{\chi}_\omega^{\pm} &= \frac{N_\omega}{\sqrt{4\pi\omega}}\Big(\frac{L+u^\pm}{L-u^\pm}\Big)^{-iL\omega/2} \\
&= \frac{N_\omega}{\sqrt{4\pi\omega}} \times \begin{cases} \exp(-i\omega U^\pm), & \text{for } |u^\pm| < L \\ e^{-\pi L\omega/2}\exp(-i\omega U^\pm_{\mathrm{ex}}), & \text{for } |u^\pm| > L \end{cases} \\
N_\omega &= (1-e^{-\pi L\omega})^{-1/2},
\end{aligned}
\tag{1}
$$

where the null coordinates $U^\pm_{\mathrm{ex}}$ are defined by

$$
u^\pm = L\coth\frac{U^\pm_{\mathrm{ex}}}{L}
$$

for the regions: $|u^\pm| > L$. Although there are two options to extend $\chi_\omega^{O\pm}$ to $|u^\pm| > L$ corresponding to the double signs in the relation $\log(-1) = \pm i\pi$, we remove this ambiguity by requiring that $\widetilde{\chi}_\omega^{\pm}$ correspond to the positive frequency modes with respect to the Poincaré invariant vacuum.

The positive frequency modes complement to $\{\chi_\omega^{O\pm}\}$ are determined as

$$
\chi_\omega^{\mathrm{ex}\pm} = \frac{1}{\sqrt{4\pi\omega}}\exp(i\omega U^\pm_{\mathrm{ex}})\theta(|u^\pm| - L),
$$

where we set the sign in the exponent to positive for $U^\pm_{\mathrm{ex}}$ are past-directed. The analytic extension of $\chi_\omega^{\mathrm{ex}\pm}$ from the regions: $|u^\pm| > L$ to M is obtained in the form

$$
\widetilde{\chi}_\omega^{\mathrm{ex}\pm} = \frac{N_\omega}{\sqrt{4\pi\omega}} \times \begin{cases} e^{-\pi L\omega/2}\exp(i\omega U^\pm), & \text{for } |u^\pm| < L \\ \exp(i\omega U^\pm_{\mathrm{ex}}), & \text{for } |u^\pm| > L. \end{cases}
\tag{2}
$$

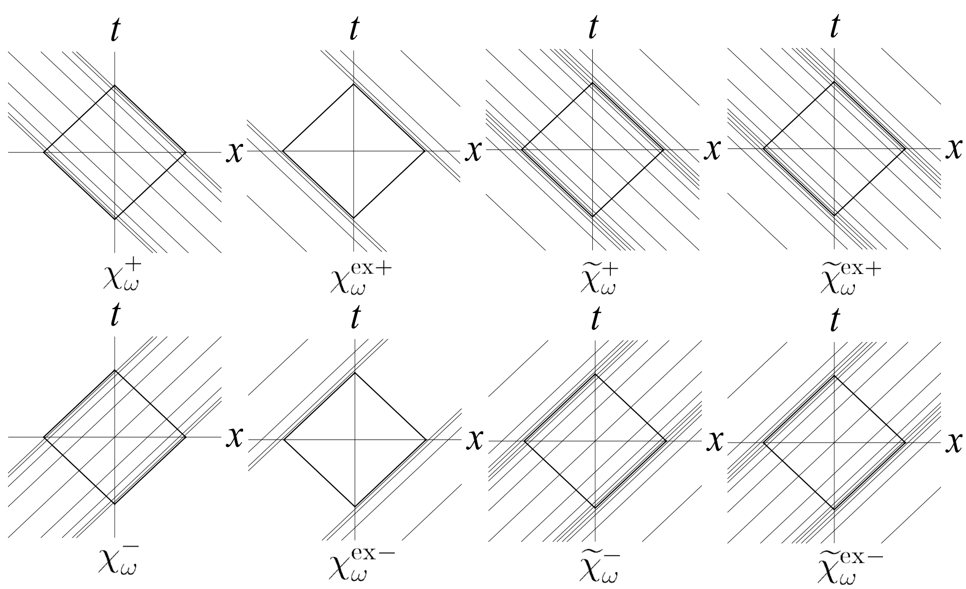

FIGURE 2. For two sets of mode functions $(\chi_\omega^\pm, \chi_\omega^{\mathrm{ex}\pm})$ and $(\widetilde{\chi}_\omega^\pm, \widetilde{\chi}_\omega^{\mathrm{ex}\pm})$, the constant phase lines are schematically depicted.

Now, let us derive the Bogoliubov transformation between the Poincaré invariant vacuum and the vacuum defined by the conformal time flow in the diamond region O.

We can expand the field operator in the form

$$\phi = \int_0^\infty d\omega (b_\omega^+ \chi_\omega^+ + b_\omega^- \chi_\omega^- \\ + b_\omega^{\mathrm{ex}+} \chi_\omega^{\mathrm{ex}+} + b_\omega^{\mathrm{ex}-} \chi_\omega^{\mathrm{ex}-} + \mathrm{H.c.}).$$

Then, the vacuum state in the diamond region $|0;O\rangle$ is defined in terms of the annihilation operators $(b_\omega^\pm, b_\omega^{\mathrm{ex}\pm})$ as

$$b_\omega^\pm |0;O\rangle = b_\omega^{\mathrm{ex}\pm}|0;O\rangle = 0.$$

On the other hand, the Poincaré invariant vacuum is defined by the set of modes $(\widetilde{\chi}_\omega^\pm, \widetilde{\chi}_\omega^{\mathrm{ex}\pm})$. More precisely, by writing the mode expansion of the field operator as

$$\phi = \int_0^\infty d\omega (a_\omega^+ \widetilde{\chi}_\omega^+ + a_\omega^- \widetilde{\chi}_\omega^- \\ + a_\omega^{\mathrm{ex}+} \widetilde{\chi}_\omega^{\mathrm{ex}+} + a_\omega^{\mathrm{ex}-} \widetilde{\chi}_\omega^{\mathrm{ex}-} + \mathrm{H.c.}),$$

the Poincaré invariant vacuum $|0;M\rangle$ is defined by

$$a_\omega^\pm |0;M\rangle = a_\omega^{\mathrm{ex}\pm}|0;M\rangle = 0.$$

From Eqs. (1) and (2), the transformation between the two sets of mode functions turns out to be

$$\widetilde{\chi}_\omega^\pm = N_\omega \left(\chi_\omega^\pm + e^{-\pi L\omega/2} (\chi_\omega^{\mathrm{ex}\pm})^* \right),$$
$$\widetilde{\chi}_\omega^{\mathrm{ex}\pm} = N_\omega \left(\chi_\omega^{\mathrm{ex}\pm} + e^{-\pi L\omega/2} (\chi_\omega^{\pm})^* \right).$$

This leads to the Bogoliubov transformation of the creation and annihilation operators as

$$a_\omega^\pm = N_\omega \left(b_\omega^\pm - e^{-\pi L\omega/2} b_\omega^{\mathrm{ex}\pm\dagger} \right),$$
$$a_\omega^{\mathrm{ex}\pm} = N_\omega \left(b_\omega^{\mathrm{ex}\pm} - e^{-\pi L\omega/2} b_\omega^{\pm\dagger} \right).$$

From this, we see that the vacuum $|0;M\rangle$ is also written formally as

$$|0;M\rangle = Z^{-1/2} \\ \times \prod_\omega \exp[e^{-\pi L\omega/2}(b_\omega^{+\dagger} b_\omega^{\mathrm{ex}+\dagger} + b_\omega^{-\dagger} b_\omega^{\mathrm{ex}-\dagger})]|0;O\rangle,$$
$$Z = \prod_\omega (1 - e^{-\pi L\omega})^{-2}.$$

By taking the partial trace of the density operator over the subsystem generated by the operators $b_\omega^{\mathrm{ex}\pm\dagger}$, we obtain the Gibbs state with the inverse temperature $\beta = \pi L$ as

$$\rho^O = Z^{-1} e^{-\pi L H^O},$$
$$H^O = \int_0^\infty d\omega\ \omega (b_\omega^{+\dagger} b_\omega^+ + b_\omega^{-\dagger} b_\omega^-).$$

It should be noted, however, that there is an ambiguity in the normalization of the conformal Killing vector field, $\xi^\mu \mapsto \alpha \xi^\mu$, which affects the inverse temperature

as $\beta \mapsto \alpha^{-1}\beta$. The invariant inverse temperature is given by $\beta^O = \beta\sqrt{-\xi_\mu \xi^\mu}$, which is regarded as the local inverse temperature associated with the observer following the flow determined by the conformal Killing vector. In terms of the proper time τ of the observer, it becomes

$$\begin{aligned}\beta^O &= \frac{2\pi}{La^2}(\sqrt{1+a^2L^2} - \cosh(a\tau)),\ \tau \in (-\tau_a, \tau_a),\\ \tau_a &= a^{-1}\mathrm{arcsinh}(aL),\end{aligned}$$

where a denotes the proper acceleration of the observer and $2\tau_a$ is the proper length of his lifetime. This is identical with the diamond temperature of Martinetti and Rovelli [3].

Thus, we come to the same conclusion with different independent arguments, which is the evidence that the diamond temperature has the universal significance. We also note that the tunneling approach recently proposed by Banerjee and Majhi [10, 11], which is another independent method to obtain the temperature of the subsystem, also gives the same temperature, though we don't state details here.

To promote a better understanding of the thermodynamics of an observer in O, we try to find the expression for the energy and the entropy in the diamond region. Firstly, we compute the expectation value of the stress-energy operator $T^O_{\mu\nu}$ for the observer in O with respect to the Poincaré invariant vacuum. The operator $T^O_{\mu\nu}$ is defined in O by

$$T^O_{\mu\nu} =: \phi_{,\mu}\phi_{,\nu} : -\frac{1}{2}g^O_{\mu\nu} : \phi^{,\alpha}\phi_{,\alpha} :,$$

where the colons denote normal ordering with respect to the vacuum $|0;O\rangle$, and the metric $g^O_{\mu\nu}$ in O is written as

$$\begin{aligned}g^O &= \frac{-dT^2 + dX^2}{\cosh^2(U^+/L)\cosh^2(U^-/L)},\\ T &= \frac{U^+ + U^-}{2},\ X = \frac{U^+ - U^-}{2}.\end{aligned} \tag{3}$$

Noting that the Hamiltonian H^O defining the present thermal state can be written as the spatial integral of T^O_{TT}, we formally obtain its expectation value as

$$\begin{aligned}\langle 0;M|H^O|0;M\rangle &= \int_{-\infty}^{\infty} dX\,\langle 0;M|T^O_{TT}|0;M\rangle\\ &= \delta(0)\int_0^\infty d\omega \frac{2\omega}{e^{\pi L\omega}-1} = \frac{\delta(0)}{3L^2}.\end{aligned}$$

The presence of the divergent factor $\delta(0)$ is typical for the quantum field, and it can be regularized by introducing a certain cut-off scale, if necessary.

On the other hand, the von Neumann entropy $S^O(\rho^O)$ of the Gibbs state ρ^O can be formally computed as

$$\begin{aligned}S^O(\rho^O) &= -\mathrm{Tr}^O(\rho^O\log\rho^O) = \pi L\langle 0;M|H^O|0;M\rangle + \log Z\\ &= \delta(0)\left[\frac{\pi}{3L} + \int_0^\infty d\omega\,\log(1-e^{-\pi L\omega})^{-2}\right]\\ &= \frac{2\pi}{3L}\delta(0),\end{aligned}$$

where the trace is taken over the Fock space of the creation and annihilation operators $(b_\omega^{\pm\dagger}, b_\omega^{\pm})$. This may be called as the entanglement entropy of the diamond region O.

In general, the amount of the entropy to energy ratio S/E contained within a given finite region is believed to be bounded from above by the typical length scale R of the region as

$$S/E < 2\pi R.$$

This is known as the Bekenstein bound [12]. In the present case, we find the relationship

$$S^O(\rho^O) = 2\pi L \langle 0; M | H^O | 0; M \rangle$$

among the entropy $S^O(\rho^O)$, the length scale L and the energy $\langle 0; M | H^O | 0; M \rangle$. This shows that the present system saturates the Bekenstein bound.

Finally, let us try to speculate on another interpretation of the entropy $S^O(\rho^O)$ in terms of the information theory. The trajectory of the modular flow is the curve: $X = \text{const.}$, which corresponds to the uniformly accelerated motion with the proper acceleration $a = -L^{-1}\sinh(2X/L)$. In other words, each congruence class of trajectories of the modular flow under the action of the proper Poincaré group is represented by the pair of parameters (L, X). Each trajectory of the modular flow defines a nonnegative function

$$P(L, X; T) = \frac{1}{2L}\frac{du^+(T)}{dT} = \frac{1}{2L\cosh^2\left(\frac{X+T}{L}\right)}$$

of the modular parameter T on the trajectory, which integrates to unity:

$$\int_{-\infty}^{\infty} dT\, P(L, X; T) = 1.$$

We interpret this as determining a certain probability density associated with the modular flow. For example, if an observer (L, X) following the modular flow regards the increase of the Minkowski time u^+ as a probabilistic process, so that u^+ jumps from $-L$ to L once in his history at the modular time T, he could expect that this jump occurs with the probability density $P(L, X; T)$.

Given the family of probability density functions $P(L, X; T)$, the parameter space (L, X) inherits the structure of the Riemannian manifold. The Riemannian metric on the parameter space is given by the Fisher information metric

$$G_{ij}(L, X) = -\int_{-\infty}^{\infty} dT\, P(L, X; T)\frac{\partial^2}{\partial y^i \partial y^j}\log P(L, X; T),$$

where $y^i = (L, X)$ denotes the coordinates on the parameter space. In the present case, the parameter space (L, X) turns out to be the Poincaré half plane. In fact, the Fisher information metric has the form

$$G = \frac{(1 + 2\zeta(2))dL^2 + 4dX^2}{3L^2}.$$

The distance in the parameter space determined by the Fisher information metric gives an invariant measure of the difference between a pair of probability density

functions. Applying this to the most distant pair: $P = (L, -\infty)$ and $Q = (L, \infty)$ in the diamond region O of the fixed size L, we get

$$\mathrm{Dist}(P, Q) = \int_{-\infty}^{\infty} \frac{2}{\sqrt{3}L} dX = \frac{4\pi}{\sqrt{3}L}\delta(0) = 2\sqrt{3}S^O(\rho^O).$$

Thus, this amount of information discrepancy is proportional to the entanglement entropy.

We can also compute explicitly the Shannon entropy $S^{\mathrm{Sh}}(L)$ as

$$S^{\mathrm{Sh}}(L) = \int_{-\infty}^{\infty} dT P(L, X; T) \log \frac{1}{P(L, X; T)} = \log \frac{e^2 L}{2},$$

which is a function of L. This quantity can be also related with the distance between the point $P = (L, X)$ and $P' = (L + \delta L, X)$ as

$$\begin{aligned}\mathrm{Dist}(P, P') &= \int_{L}^{L+|\delta L|} \sqrt{\frac{1 + 2\zeta(2)}{3}} \frac{dL}{L} \\ &= \sqrt{\frac{1 + 2\zeta(2)}{3}} [S^{\mathrm{Sh}}(L + |\delta L|) - S^{\mathrm{Sh}}(L)].\end{aligned}$$

Thus, the Shannon entropy is relevant to the entropy correction [13, 14] associated with the variation of the size of the diamond region O.

In this way, the Riemannian structure of the parameter space of a certain kind of the probability density functions might have to do with the entanglement entropy of the subsystem and its corrections. We hope this viewpoint provides some insight into the better understanding of the information-theoretic origin of the Bekenstein-Hawking entropy of black holes.

References

[1] W. Driessler, S. J. Summers and E. H. Wichmann, Commun. Math. Phys. **105**, 49 (1986).
[2] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras* (Springer, Berlin, 1996).
[3] P. Martinetti and C. Rovelli, Classical Quantum Gravity **20** 4919 (2003).
[4] S. A. Fulling, Phys. Rev. D **7**, 2850 (1973); P. C. W. Davies, J. Phys. A **8**, 609 (1975); W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[5] J. J. Bisognano and E. H. Wichmann, J. Math. Phys. **16**, 985 (1975).
[6] M. Takesaki, *Tomita's Theory of Modular Hilbert Algebras and its Applications*, Lecture Notes in Mathematics **128** (Springer, Berlin, 1970).
[7] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (W. A. Benjamin, Inc., New York, 1964).
[8] P. D. Hislop and R. Longo, Commun. Math. Phys. **84**, 71 (1982).
[9] See e.g., N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, Cambridge, 1984).
[10] M. K. Parikh and F. Wilczek, Phys. Rev. Letters **85**, 5042 (2000).
[11] R. Banerjee and B. R. Majhi, Phys. Lett. B **675**, 243 (2009).
[12] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981).
[13] L. Susskind and J. Uglum, Phys. Rev. D **50**, 2700 (1994).
[14] C. Callan and F. Wilczek, Phys. Lett. B **333**, 55 (1994).